A. G. Kofman[1], G. Kurizki[1] and T. Opatrný[1,2]

[1] *Department of Chemical Physics, Weizmann Institute of Science, 761 00 Rehovot, Israel*

[2] *Theoretisch Physikalisches Institut, Friedrich-Schiller Universität, Max-Wien-Platz 1, D-0743 Jena, Germany, and Department of Theoretical Physics, Palacký University, Svobody 26, CZ-77146 Olomouc, Czech Republic*

We discuss a simple, experimentally feasible scheme, which elucidates the principles of controlling ("engineering") the reservoir spectrum and the spectral broadening incurred by repeated measurements. This control can yield either the inhibition (Zeno effect) or the acceleration (anti-Zeno effect) of the quasi-exponential decay of the observed state by means of frequent measurements. In the discussed scheme, a photon is bouncing back and forth between two perfect mirrors, each time passing a polarization rotator. The horizontal and vertical polarizations can be viewed as analogs of an excited and a ground state of a two level system (TLS). A polarization beam splitter and an absorber for the vertically polarized photon are inserted between the mirrors, and effect measurements of the polarization. The polarization angle acquired in the electrooptic polarization rotator can fluctuate randomly, e.g., via noisy modulation. In the absence of an absorber the polarization randomization corresponds to TLS decay into an *infinite-temperature reservoir*. The non-Markovian nature of the decay stems from the many round-trips required for the randomization. We consider the influence of the polarization measurements by the absorber on this non-Markovian decay, and develop a theory of the Zeno and anti-Zeno effects in this system.

PACS numbers: 03.65.Bz, 42.50.-p, 03.67.-a, 42.25.Hz

I. INTRODUCTION

The quantum Zeno effect (QZE) is the striking prediction that the decay of *any* unstable quantum state can be inhibited by sufficiently frequent observations (measurements) [1–3]. The QZE has been experimentally tested [4] and primarily analyzed for *two coupled states* [5–7] (with few exceptions [8,9]). Yet the consensus opinion has upheld the QZE as a *general* feature of quantum mechanics which should lead, e.g., to the inhibition of radioactive or radiative decay [10,11]. The claim of QZE generality has rested on the assumption that successive observations can, *in principle*, always be made at time intervals too short for the system to change appreciably. However, this assumption and the generality of the QZE have scarcely been investigated.

We have now shown [12] that this assumption is basically incorrect and that the QZE does not hold generally, but only in a restricted class of systems. The main implications of our theory are: (i) The QZE is principally unattainable in radiative or radioactive decay, because the required measurement rates may cause the system to disintegrate (via the production of new particles). (ii) Decay *acceleration* by frequent measurements (the anti-Zeno effect - AZE) [12–15] is possible for essentially *any* decay process, and is thus much more ubiquitous than its inhibition (the QZE). These findings stem from the *universal* result [12], whereby the modification of the decay rate by frequent measurements is determined by the convolution (overlap) of two functions: (a) the measurement-induced spectral broadening (*energy spread*), which is proportional to the rate of measurements, in accordance with the time-energy uncertainty relation; (b) the *spectrum of the reservoir* (bath) to which the decaying state is coupled. The QZE or AZE correspond to the measurement-induced spread being much broader or narrower than the reservoir spectral width, respectively. The *non-Markovian nature of any physical decay process*, associated with the *finite* spectral width (and, correspondingly, nonzero memory time) of the reservoir, is the essential property allowing the modification of the decay by means of frequent measurements, be it the QZE or the AZE.

The universal formula [12] was obtained on the basis of the projection postulate. In reality there can be many different measurement schemes, which can be classified as direct, when the initial state itself is measured [4,6,7,9], and indirect [11,16,17], when the final state(s) are measured. Whereas direct measurements should be nondestructive, i.e., conserving the population of the measured state, indirect measurements can be either nondestructive or destructive. This difference can affect the dynamics for large times, when the initial-state population significantly differs from 1. However, for short times, when this population is close to one, formula obtained in [12] should hold for the both types of measurements.

In this paper we discuss a simple, experimentally feasible scheme, which elucidates the principles of controlling ("engineering") the reservoir spectrum and the spectral broadening incurred by repeated measurements. This control can yield either the inhibition (QZE) or the acceleration (AZE) of the quasi-exponential (non-Markovian) decay of the observed state by means of frequent measurements. In the present scheme, the pertinent observable is photon polarization, constituting the optical analog of a two-level system [16]. The photon is injected into the setup via a fast gate, which first reflects and then transmits horizontal polarization. The injected photon is bouncing back and forth between two perfect mirrors, each time passing a polarization rotator (Fig. 1) [18]. A polarization beam splitter (PBS), which reflects a vertically polarized photon and transmits a horizontally polarized one, as well as an absorber, are inserted between the mirrors. If the path of the vertically polarized photon is completely blocked by the absorber, then it realizes a discrete measurement (projection) at each passage: the photon is either lost or its state is projected onto the horizontal polarization. This situation has been used to demonstrate an interaction-free measurement [19–21]. If, on the other hand, the absorber is partially transparent to vertically polarized photons, then it realizes an imperfect measurement [22,23].

The polarization angle acquired in the electrooptic polarization rotator can fluctuate randomly, e.g., via noisy modulation of a Pockels cell. In the absence of the absorber, the polarization, after *many round-trips,* then becomes random and the probability of finding any particular polarization tends to 1/2. Taking the horizontal and vertical polarizations as analogs of an excited and a ground state of a two level system, this polarization randomization corresponds to decay into an *infinite-temperature reservoir*. The non-Markovian nature of the decay stems from the many round-trips required for the polarization to change randomly, which implies a *long memory time*. Our goal is to consider the influence of the polarization measurements by the absorber on this non-Markovian decay, and derive the conditions of the QZE and AZE in this system. It should be noted that if the source of injected photons is a laser, governed by quasiclassical photon statistics, then the results of polarization decay can be interpreted classically. However, the concept of *the Zeno effect is valid in classical electromagnetism*, as pointed out by Peres [16].

In Sec. II we present the physical model and its features in the limits of perfect absorption or fixed (rather than random) rotation angles. In Sec. III the master equations for its general analysis are derived (any absorption and rotation). Sec. IV is devoted to continuous dephasing, i.e., the limit of small, highly correlated random rotations and weak absorption (ineffective measurements). Sections V and VI deal with discrete dephasing (namely, correlated and anticorrelated phase jumps) and arbitrary absorption (effective and ineffective measurements). Conditions for the QZE and AZE are derived. In particular, in Sec. V a general theory is developed for the most interesting case of *small rotation angles*, whereas in Sec. VI a simple non-Markovian model for random rotations of an *arbitrary size* is studied analytically and numerically. The conclusions are given in Sec. VII. Appendices A, B and C give the details of the general analysis, the small and the arbitrary-size phase-jump analysis, respectively.

II. MODEL DESCRIPTION

A. The setup

Consider the setup in Fig. 1 [18]. A horizontally polarized photon, denoted as $|h\rangle$, enters the setup via a fast gate (not shown), which changes from being totally reflective to totally transparent to $|h\rangle$ on a ns scale. The polarization rotator between the two highly reflecting mirrors causes fast (ns-scale) rotation of the photon polarization by means of a Pockels cell or another electrooptic element. The $|h\rangle$ photon, transmitted by the PBS, bounces between the mirrors, while a vertically polarized photon, denoted as $|v\rangle$, which is reflected by the PBS, is blocked by an absorber, which can be made *partially transparent* with transmissivity θ. (Alternatively, one can use a perfect absorber and a PBS which is partially transparent for the vertically polarized photon $|v\rangle$.)

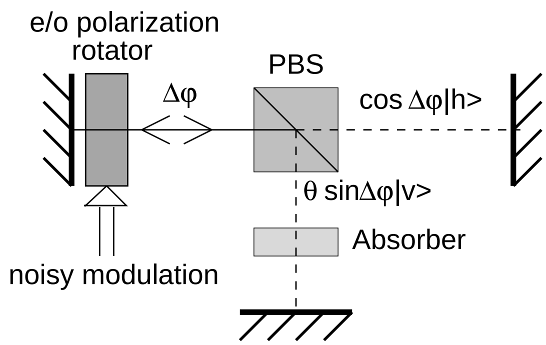

FIG. 1. Setup for controlling the polarization decay of a photon bouncing between the mirrors. Measurements are effected by a polarization beam-splitter (PBS) and an absorber with variable transparency θ. The “reservoir” into which the polarization decays is realized upon modulating a Pockels cell (which rotates the polarization by $\Delta\varphi$) by a field with controllable noise properties.

B. QZE for constant rotation angle and complete absorption

When the path of $|v\rangle$ is open, the probability of finding the photon with horizontal polarization after n round trips is

$$P_h(n) = \cos^2(n\Delta\varphi), \tag{1}$$

assuming a fixed $\Delta\varphi$, the rotation angle of the polarizer at each passage: the polarization oscillates between the vertical and horizontal states, analogously to a Rabi oscillation. If, on the other hand, the path of $|v\rangle$ is completely blocked by a perfect absorber, this absorber realizes an impulsive measurement (IM), i.e. a projection, at each passage: the photon is either lost or its state is projected onto the $|h\rangle$ state. With the completely blocked $|v\rangle$ path the probability of finding $|h\rangle$ decays as

$$P_h(n) = \cos^{2n}(\Delta\varphi), \tag{2}$$

where n is the number of the round-trips. With $\Delta\varphi \ll 1$ the *effective exponential decay is much slower than the Rabi oscillation.* This constitutes an example of the QZE by IMs [4,6,12,13,21], which has been introduced as a demonstration of interaction-free measurements [19–21].

C. Arbitrary absorption and rotation

To extend the above results to the case of partial absorption and arbitrary rotation angle $\Delta\varphi_k$ ($k = 1, \ldots$), we write down the obvious dynamical equations for the horizontal and vertical components of the photon field envelope (wave function), ϵ_h and ϵ_v, respectively,

$$\begin{aligned}\dot{\epsilon}_h &= -\mu(t)\epsilon_v,\\ \dot{\epsilon}_v &= \mu(t)\epsilon_h - \frac{\Gamma(t)}{2}\epsilon_v.\end{aligned} \tag{3}$$

Here $\mu(t) = \dot{\varphi}(t)$, $\varphi(t)$ is the total angle accumulated by consecutive $\Delta\varphi_k$ between the polarization vector and the vertical axis of the polarizer, and $\Gamma(t)$ is the photon absorption rate in each passage. The rates of polarization rotation and absorption, $\mu(t)$ and $\Gamma(t)$, are time-periodic functions, whose period is the round-trip time $\tau_{\rm r}$. They do not temporally overlap in the setup of Fig. 1, i.e., there exists τ_1 ($\tau_1 < \tau_{\rm r}$) such that $\mu(t)$ vanishes for $\tau_1 \le t \le \tau_{\rm r}$ while $\Gamma(t)$ vanishes for $0 \le t \le \tau_1$. Equation (3) is equivalent to the Schrödinger equation for an *open two-level system* (TLS) *interacting with a field* $\mu(t)$.

The general solution of Eq. (3) at $t = n\tau_{\rm r}$ is

$$\begin{pmatrix}\epsilon_h(t)\\ \epsilon_v(t)\end{pmatrix} = \left[\prod_{k=1}^{n}\begin{pmatrix}\cos\Delta\varphi_k & -\sin\Delta\varphi_k\\ \theta\sin\Delta\varphi_k & \theta\cos\Delta\varphi_k\end{pmatrix}\right]\begin{pmatrix}\epsilon_h(0)\\ \epsilon_v(0)\end{pmatrix}, \tag{4}$$

where

$$\theta = \exp\left[-\frac{1}{2}\int_0^{\tau_{\rm r}}\Gamma(t)dt\right] \tag{5}$$

is the amplitude transmission coefficient of the absorber. Within the above constraints $\mu(t)$ and $\Gamma(t)$ are *arbitrary.* Most of the subsequent analysis will be dedicated to *random* $\mu(t)$ caused by noisy modulation of the electro-optic rotator.

D. QZE for fixed rotation angle and incomplete absorption

Assuming again a *fixed* $\Delta\varphi$ (all $\Delta\varphi_k$ being equal), one obtains from (4) the following time-dependent probability for the photon to keep its initial horizontal polarization ($\epsilon_h(0) = 1$, $\epsilon_v(0) = 0$)

$$\begin{aligned}P_h(t = n\tau_{\rm r}) &= \epsilon_h^2(t)\\ &= \{[\lambda_1^n(\cos\Delta\varphi - \lambda_2) + \lambda_2^n(\lambda_1 - \cos\Delta\varphi)]/D\}^2,\end{aligned} \tag{6}$$

where $D = \sqrt{(1+\theta)^2\cos^2\Delta\varphi - 4\theta}$ and $\lambda_{1,2} = [(1+\theta)\cos\Delta\varphi \pm D]/2$. For complete transparency, $\theta = 1$, Eq. (6) reduces to the result (1) and for complete absorption, $\theta = 0$, to the result (2).

For small phase jumps $\Delta\varphi$ and sufficiently strong absorption, $(\Delta\varphi)^2 \ll (1-\theta)^2$, the photon polarization decay is approximately exponential

$$P_h(t = n\tau_{\rm r}) = \exp\left[-\frac{(1+\theta)(\Delta\varphi)^2}{(1-\theta)\tau_{\rm r}}t\right]. \tag{7}$$

It is intuitively clear that the effective rate of measurements ν increases with the quantity $1-\theta$, which plays the role of the effectiveness of the measurements. This will be rigorously confirmed in Sec. V. Correspondingly, the decay rate in Eq. (7) decreases with the increase of $1-\theta$, thus demonstrating the QZE in the case of fixed $\Delta\varphi$.

III. GENERAL ANALYSIS OF POLARIZATION DEPHASING: MASTER EQUATIONS

In the ensuing analysis we assume that the *rotation angle* $\Delta\varphi$ *is not fixed but fluctuates randomly*, e.g., due to the modulation of the Pockels cell rotator by a *noisy control field.* Thus, after many round trips the polarization of the photon becomes random and the probability of finding any particular polarization tends to 1/2. Taking the horizontal and vertical polarizations as analogs of the excited and ground states of a two level system (TLS), *this polarization randomization corresponds to decay into an infinite-temperature reservoir.*

In what follows, we shall write down the most general equations describing the influence of polarization projection measurements on such decay. The polarization tensor (or the density matrix for the TLS) obeys the equation

$$\dot{Q} = [A(t) + C\mu(t)]Q. \tag{8}$$

Here $Q = (P_h, P_v, u)^\dagger$, $P_v(t = n\tau_r) \equiv \epsilon_v^2(t)$ is the probability for the photon to have the vertical polarization, $u = 2\epsilon_v\epsilon_h$ is the coherence, in TLS terms, or the first Stokes parameter [24], and

$$A(t) = \begin{pmatrix}0 & 0 & 0\\ 0 & -\Gamma(t) & 0\\ 0 & 0 & -\frac{\Gamma(t)}{2}\end{pmatrix}, \quad C = \begin{pmatrix}0 & 0 & -1\\ 0 & 0 & 1\\ 2 & -2 & 0\end{pmatrix}. \tag{9}$$

Equation (8) can be reduced to an equation for the average quantity $\bar{Q}(t)$, involving an expansion in cumulants of $C\mu(t)$ [25,26]. As shown in Appendix A, by truncating the cumulant expansion at the second order, one obtains the following *non-Markovian* differential master equations for the average polarization probabilities,

$$\frac{d\bar{P}_h}{dt} = -R(t)\bar{P}_h + R(t)\bar{P}_v,$$
$$\frac{d\bar{P}_v}{dt} = R(t)\bar{P}_h - [R(t)+\Gamma(t)]\bar{P}_v. \quad (10)$$

The rate that governs the polarization change in (10) is given by the integral

$$R(t) = 2\int_0^t dt' k(t,t')\theta(t,t'), \quad (11)$$

whose integrand is the product of

$$k(t,t') = \langle \mu(t)\mu(t')\rangle, \quad (12)$$

the correlation function of the random rotation rate $\mu(t)$, and of

$$\theta(t,t') = \exp\left[-\frac{1}{2}\int_{t'}^t \Gamma(t_1)dt_1\right], \quad (13)$$

which is related to the polarizer transparency (measurement effectiveness, as discussed below). We assume that the average of $\mu(t)$ vanishes, $\overline{\mu(t)} = 0$ (no systematic shift $\Delta\varphi$).

To obtain the validity condition of the above master equations, one should consider higher-order terms in the cumulant expansion. As shown in Appendix A for the case of continuous noise, the comparison of the second and fourth cumulants implies that Eqs. (10) hold under the condition

$$R \ll \Gamma_{\mathrm{R}}, \quad (14)$$

where Γ_{R} is the reciprocal correlation time of $\mu(t)$. Below we assume that criterion (14) holds also for the case of discrete noise.

IV. CONTINUOUS DEPHASING

In this section we consider the case when $\bar{P}_{h,v}(t)$ *vary slowly on the time scale of several round trips*, which allows one to describe them by continuous functions of time on the coarse-grained time scale (a more general case will be discussed in Sec. V). In the present case the phase jumps

$$\Delta\varphi_n = \int_{(n-1)\tau_{\mathrm{r}}}^{n\tau_{\mathrm{r}}} dt\mu(t) \quad (15)$$

are small, $B^2 \equiv \langle(\Delta\varphi_n)^2\rangle \ll 1$, and highly correlated, $\Delta\varphi_n \approx \Delta\varphi_{n-1}$, whereas the measurements are highly non-effective, $\theta \approx 1$. Then $\Gamma(t)$ can be substituted by

$$\Gamma_0 = \frac{1}{\tau_{\mathrm{r}}}\int_0^{\tau_{\mathrm{r}}} dt\Gamma(t), \quad (16)$$

whereas $\mu(t)$ can be considered as a continuous, stationary random process, implying $k(t,t') = k(t-t')$. As a result, now in Eqs. (10)

$$R(t) = 2\int_0^t dt' k(t')\exp\left(-\frac{\Gamma_0 t'}{2}\right). \quad (17)$$

For $t \gg \Gamma_{\mathrm{R}}^{-1}$, where Γ_{R} is the characteristic decay rate of $k(t)$, Eqs. (10) become Markovian (although they account for non-Markovian phase fluctuations associated with $\mu(t)$)

$$\frac{d\bar{P}_h}{dt} = -R\bar{P}_h + R\bar{P}_v,$$
$$\frac{d\bar{P}_v}{dt} = R\bar{P}_h - (R+\Gamma_0)\bar{P}_v, \quad (18)$$

with *constant rate*

$$R = 2\int_0^\infty dt k(t)\exp\left(-\frac{\Gamma_0 t}{2}\right). \quad (19)$$

The last factor in the integrand here expresses the decay law of the first Stokes parameter, $f(t) = \bar{u}(t)/u(0)$, due to measurement-induced relaxation. This follows from Eqs. (8) [with $\mu(t) = 0$] and (16).

The solution of Eqs. (18) with the initial conditions

$$\bar{P}_h(0) = 1, \qquad \bar{P}_v(0) = 0, \quad (20)$$

yields

$$\bar{P}_h(t) = e^{-(R+\Gamma_0/2)t}\left(\cosh St + \frac{\Gamma_0}{2S}\sinh St\right), \quad (21)$$

where $S = \sqrt{R^2 + \Gamma_0^2/4}$.

In the absence of measurements, $\Gamma_0 = 0$, (21) yields

$$\bar{P}_h(t) = \tfrac{1}{2}(1 + e^{-2R_0 t}), \quad (22)$$

where $R_0 = 2\int_0^\infty dt k(t)$. By contrast, the measurement-affected polarization decays, assuming that $\Gamma_0 \gg R$, as

$$\bar{P}_h(t) \approx e^{-Rt}, \quad (23)$$

so that R is indeed the measurement-modified decay rate.

The expression for the decay rate (19) can be recast in the same form as the universal result in Ref. [12],

$$R = 2\pi\int_{-\infty}^\infty G(\omega)F(\omega)d\omega. \quad (24)$$

Here

$$G(\omega) = \frac{1}{\pi}\int_0^\infty dt k(t)\cos\omega t \quad (25)$$

is the random-field intensity spectrum, which can be considered as the spectrum of the *infinite-temperature reservoir*. The other factor in (24),

$$F(\omega) = \frac{1}{\pi}\frac{\Gamma_0/2}{(\Gamma_0/2)^2 + \omega^2}, \quad (26)$$

is the Fourier transform of the measurement-induced decay law of the first Stokes parameter. The width of $F(\omega)$

has the meaning of the effective rate of measurements ν [12]. The definition of ν introduced in [12] becomes in the present case of a degenerate TLS

$$\nu = [\pi F(0)]^{-1}. \tag{27}$$

From Eq. (26) one obtains $\nu = \Gamma_0/2$.

Equation (24) allows a graphical interpretation of the QZE [14,27]. In particular, Eq. (24) shows that the *QZE occurs when the reservoir spectrum is peaked around* $\omega = 0$. However if *the spectral peak of the reservoir is detuned from the resonance frequency* ω_a of the TLS (here $\omega_a = 0$), one can obtain the quantum anti-Zeno effect (AZE), i.e., an increase of the decay rate R with the effective measurement rate ν, as illustrated in Sec. V A.

V. DISCRETE DEPHASING (PHASE JUMPS)

A. Small jumps: General analysis

Here we allow for any degree of correlation between consecutive phase jumps, as well as for arbitrary absorption per passage. In this subsection a general theory is developed for the case of sufficiently small phase jumps.

We consider the sequence of polarization rotations by the angles $\Delta\varphi_n$ to be a discrete-time random process with the correlation function $K_{nm} = \langle \Delta\varphi_n \Delta\varphi_m \rangle$. Typically, the random process $\Delta\varphi_n$ is *stationary*, yielding $K_{nm} = K_{n-m} = K_{m-n}$. The general analysis is given in Appendix B. Here we present the simple case

$$K_n = B^2 \gamma^{|n|}, \tag{28}$$

where γ ($-1 \le \gamma \le 1$) is the correlation degree between two successive jumps [26]: $\Delta\varphi_n \approx \Delta\varphi_{n+1}$ for $\gamma \approx 1$, $\Delta\varphi_n$ and $\Delta\varphi_{n+1}$ tend to have opposite signs for $\gamma < 0$ and are statistically independent for $\gamma = 0$. In this case [Eq. (28)] the correlation time is given by [cf. (B1)]

$$\Gamma_{\mathrm{R}}^{-1} = \frac{\tau_{\mathrm{r}}}{1-\gamma}. \tag{29}$$

We start from the master equations (10), which are applicable both for continuous and discrete evolution. As shown in Appendix B, under the conditions

$$n \gg \frac{|\gamma|\theta}{1-(\gamma\theta)^2}, \tag{30a}$$

$$B^2 \ll (1-\gamma)(1-\gamma\theta), \tag{30b}$$

Eqs. (10) yield solution Eq. (21), where now

$$R = \frac{1+\gamma\theta}{1-\gamma\theta}\frac{B^2}{\tau_{\mathrm{r}}}, \qquad t = n\tau_{\mathrm{r}}. \tag{31}$$

For the general case

$$R = \frac{1}{\tau_{\mathrm{r}}} \sum_{n=-\infty}^{\infty} K_n \theta^{|n|}. \tag{32}$$

One recognizes in this expression the analog of Eq. (19) for the discrete case, on realizing that θ^n ($n \ge 0$) is the discrete measurement-induced relaxation function of the first Stokes coefficient.

B. Correlated and anticorrelated discrete jumps: QZE and AZE

The decay rate R (31) is independent of the measurements for uncorrelated jumps, i.e., Markovian phase fluctuations ($\gamma = 0$). However, R as a function of the effectiveness of measurements $1-\theta$ decreases for correlated phase jumps ($\gamma > 0$), thus demonstrating the QZE, and increases for anticorrelated jumps ($\gamma < 0$), demonstrating the AZE (Fig. 2). How can we interpret these results?

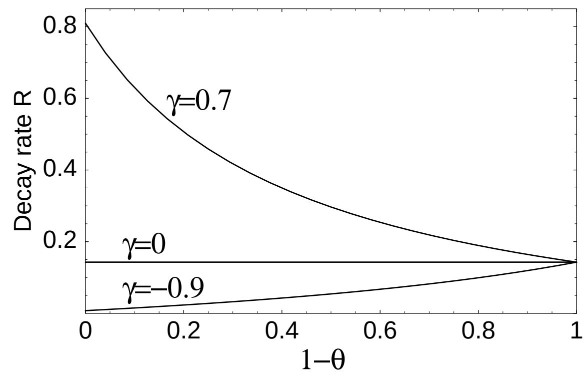

FIG. 2. Decay rate R dependence on measurement effectiveness $1-\theta$ for different degrees of correlation between consecutive phase jumps, according to Eqs. (31) and (33) Here $B = 0.1$, $\tau_{\mathrm{r}} = 0.07$. For the curves from top to bottom $\gamma = 0.7$ (correlation leading to QZE), 0, -0.9 (anticorrelation leading to AZE).

The first step towards gaining insight into the decay rate (31) is to realize that it can be recast in the form (Appendix B)

$$R = 2\pi \int_{-\pi/\tau_{\mathrm{r}}}^{\pi/\tau_{\mathrm{r}}} d\omega G(\omega) F(\omega), \tag{33}$$

where

$$G(\omega) = \frac{B^2}{2\pi\tau_{\mathrm{r}}} \frac{1-\gamma^2}{1+\gamma^2-2\gamma\cos\omega\tau_{\mathrm{r}}} \tag{34}$$

is the reservoir spectrum, and

$$F(\omega) = \frac{\tau_{\mathrm{r}}}{2\pi} \frac{1-\theta^2}{1+\theta^2-2\theta\cos\omega\tau_{\mathrm{r}}} \tag{35}$$

is the Fourier transform of the measurement-induced relaxation function of the first Stokes parameter [see Eq. (B9)]. The form (33) is essentially the same as (24) or the universal form of measurement-affected decay rates given in Ref. [12]. What makes (33) distinct is that we are now in a finite frequency domain, $-\pi/\tau_\mathrm{r} < \omega < \pi/\tau_\mathrm{r}$, in contrast to Eq. (24). This is related to the fact that polarization evolution studied here is discrete in time. In what follows, we shall first separately analyze $F(\omega)$ and $G(\omega)$, and then graphically deduce the QZE and AZE from their convolution (33).

1. Properties of $F(\omega)$

The Fourier-transformed measurement-induced relaxation function of the first Stokes parameter (35) is normalized to one. It has one maximum, $F_\mathrm{max}(\omega = 0) = (\tau_\mathrm{r}/2\pi)(1+\theta)/(1-\theta)$ located at $\omega = 0$. It is minimal at the borders $\omega = \pm\pi/\tau_\mathrm{r}$ of the frequency domain, $F_\mathrm{min} = (\tau_\mathrm{r}/2\pi)(1-\theta)/(1+\theta)$. In the particular case of the ideal (fully effective) projective measurements (IMs), $F(\omega)$ is constant,

$$F(\omega) = \frac{\tau_\mathrm{r}}{2\pi} \qquad (\theta = 0), \tag{36}$$

whereas for *non-effective* (unreliable) measurements ($\theta \approx 1$), $F(\omega)$ is a narrow peak [cf. Eq. (26)],

$$F(\omega) \approx \frac{1}{\pi}\frac{\Gamma_0/2}{(\Gamma_0/2)^2 + \omega^2} \qquad (|\omega|\tau_\mathrm{r} \ll 1). \tag{37}$$

Here we took into account that now $(1-\theta) \approx \Gamma_0\tau_\mathrm{r}/2 \ll 1$ [cf. Eqs. (5) and (16)].

Inserting Eq. (35) into (27), one obtains that the effective rate of measurements is

$$\nu = \frac{2(1-\theta)}{1+\theta}\frac{1}{\tau_\mathrm{r}}. \tag{38}$$

For any allowed value of θ, we have $\nu \sim (1-\theta)/\tau_\mathrm{r}$, which formally confirms the intuitive interpretation of $1-\theta$ as the effectiveness of measurements. More specifically, for IMs, $\nu = 2/\tau_\mathrm{r}$, which differs by a factor of 2 from the real rate of measurements (note that the definition of ν is meaningful only with an accuracy up to a factor of the order of one). For low-effectiveness (highly unreliable) measurements ($\theta \approx 1$), we have $\nu = (1-\theta)/\tau_\mathrm{r} = \Gamma_0/2$.

2. Properties of $G(\omega)$

The reservoir-coupling spectrum (34) is constant in the Markovian case $G(\omega)$, $\gamma = 0$,

$$G(\omega) = \frac{B^2}{2\pi\tau_\mathrm{r}} \qquad (\gamma = 0). \tag{39}$$

By contrast, $G(\omega)$ is mainly concentrated near $\omega = 0$ for *highly correlated* jumps ($\gamma \approx 1$),

$$G(\omega) \approx \frac{B^2}{\pi\tau_\mathrm{r}^2}\frac{\Gamma_\mathrm{R}}{\Gamma_\mathrm{R}^2 + \omega^2} \qquad (|\omega|\tau_\mathrm{r} \ll 1), \tag{40}$$

whereas for highly anticorrelated jumps ($\gamma \approx -1$), it is peaked near $\omega = \pm\pi/\tau_\mathrm{r}$,

$$G(\omega) \approx \frac{B^2}{\pi\tau_\mathrm{r}^2}\frac{\Gamma'_\mathrm{R}}{{\Gamma'_\mathrm{R}}^2 + (\pi/\tau_\mathrm{r} \pm \omega)^2} \qquad (\pi \pm \omega\tau_\mathrm{r} \ll 1), \tag{41}$$

with $\Gamma'_\mathrm{R} = (1+\gamma)/\tau_\mathrm{r}$.

3. Graphical analysis of the decay rate

Since the decay rate Eq. (33) is determined by the overlap of $F(\omega)$ and $G(\omega)$, the above results allow a graphical interpretation of the dependence of R on the effective measurement rate (Fig. 3). In Fig. 3(a), $G(\omega)$ is flat ($\gamma = 0$) and the convolution (33) is proportional to the integral of $F(\omega)$, $\int_{-\pi/\tau_\mathrm{r}}^{\pi/\tau_\mathrm{r}} d\omega F(\omega) = 1$. As a result, R is independent of the shape of $F(\omega)$ (i.e., of the effectiveness of measurements). In Fig. 3(b) ($\gamma > 0$), $G(\omega)$ is peaked at $\omega = 0$ and hence the convolution R is determined by the portion of $F(\omega)$ inside the width of $G(\omega)$. This implies a reduction of R with the broadening of $F(\omega)$, i.e., the QZE. The opposite is true in Fig. 3(c), due to the fact that the reservoir peaks are detuned from the TLS frequency $\omega_a = 0$. Now the convolution R is sensitive to the wings of $F(\omega)$, which rise with the increase of $F(\omega)$. As a result, the AZE occurs (increase of R with the broadening of $F(\omega)$, i.e., with the increase of the measurement effectiveness). In Figs. 3(b), 3(c), the limit of the flat $F(\omega)$ (i.e., IMs) implies that the integral (33) is independent of the shape of $G(\omega)$, resulting in the same value of R as in the Markovian case, Fig. 3(a).

The decay rate R corresponding to Eq. (7) can be recast as

$$R = 2(\Delta\varphi/\tau_\mathrm{r})^2/\nu, \tag{42}$$

with the effective rate of measurements ν given by Eq. (38). The above expression for R has the familiar form characteristic of the QZE [12]. The coefficient $(\Delta\varphi/\tau_\mathrm{r})^2 = \bar{\mu}^2$ is the squared average of the coupling amplitude over the round trip.

VI. EXACTLY SOLVABLE MODELS OF RANDOM PHASE JUMPS

We now consider simple models allowing an exact solution in the cases of *absent measurements (free evolution)* and *perfect measurements* (IMs)].

In the case of free evolution the probability that the photon is found with the horizontal polarization is given by

$$P_h(n) = \langle\cos^2\varphi_n\rangle = \frac{1}{2} + \frac{1}{2}\mathrm{Re}\langle e^{2i\varphi_n}\rangle, \tag{43}$$

where $\varphi_n = \sum_{k=1}^{n}\Delta\varphi_k$ and the angle brackets denote the averaging.

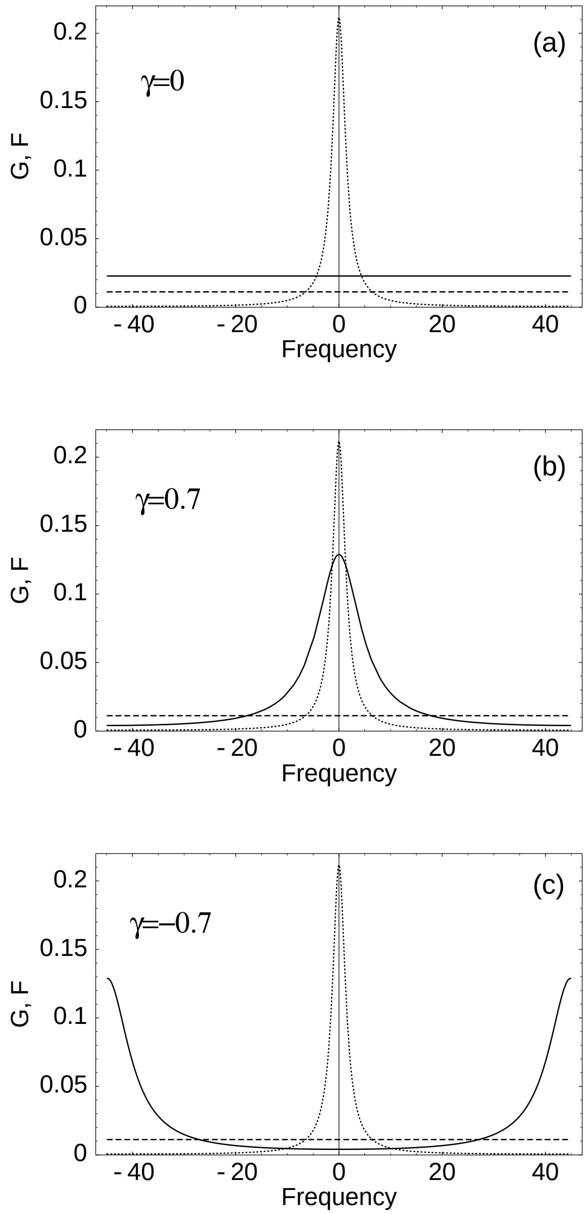

FIG. 3. Overlap of the reservoir spectrum $G(\omega)$ in (34) and the repeated-measurement broadening $F(\omega)$ in (35), determining the convolution integral of the decay rate R in (B5). Solid lines: $G(\omega)$ with (a) $\gamma = 0$ (uncorrelated jumps), (b) $\gamma = 0.7$ (correlated jumps - QZE), (c) $\gamma = -0.7$ (anticorrelated jumps - AZE). Dashed lines: $F(\omega)$ with $\theta = 0$ (perfect projections). Dotted lines: $F(\omega)$ with $\theta = 0.9$ (ineffective measurements). Here $B = 0.1$, $\tau_{\rm r} = 0.07$.

A. Markovian limit (uncorrelated jumps)

First, let us assume that the rotation angles $\Delta\varphi$ during the round-trips are independent of each other. We can then write Eq. (43) in the form

$$P_h(n) = \frac{1}{2} + \frac{1}{2}\mathrm{Re}\int p(\Delta\varphi_1)\ldots\int p(\Delta\varphi_n) \times \exp\left[2i\sum_{k=1}^{n}\Delta\varphi_k\right] d\Delta\varphi_1\ldots d\Delta\varphi_n. \tag{44}$$

Assuming the probability distribution of each $\Delta\varphi_k$ to be symmetric, we finally arrive at

$$P_h(n) = \frac{1}{2} + \frac{1}{2}\langle\cos 2\Delta\varphi_k\rangle^n, \tag{45}$$

with

$$\langle\cos 2\Delta\varphi_k\rangle = \int p(\Delta\varphi_k)\cos 2\Delta\varphi_k d\Delta\varphi_k. \tag{46}$$

If, for simplicity, we assume that the rotation angle can take just two values $\pm\Delta\varphi$, each with the probability 1/2, then Eq. (45) reduces to

$$P_h(n) = \frac{1}{2} + \frac{1}{2}\cos^n 2\Delta\varphi. \tag{47}$$

Equations (45) and (47) describe a purely exponential Markovian decay, for which we cannot expect any inhibition by the QZE.

B. Simple non-Markovian model

1. Description of the model

Let us now consider a simple model for discrete non-Markovian dephasing, in which the probability of the rotation angle in the nth step depends on the rotation angle in the $(n-1)$th step (the so-called "random walk with persistence" [28]).

We assume that the rotation step can take two values $\pm\Delta\varphi$ with equal probabilities $p_0(\pm\Delta\varphi) = 1/2$, whereas the nth rotation angle is equal or opposite to the previous one $(n-1)$ with the probability p or $q = 1-p$, respectively. Correspondingly, the conditional probabilities $P(\Delta\varphi_{n+1}, \Delta\varphi_n)$ are defined by

$$P(\Delta\varphi_n, \Delta\varphi_n) = p, \qquad P(-\Delta\varphi_n, \Delta\varphi_n) = q. \tag{48}$$

As a consequence, for a given $\Delta\varphi_n$ the conditional average of $\Delta\varphi_{n+1}$ is

$$\langle\Delta\varphi_{n+1}\rangle_{\Delta\varphi_n} = \gamma\Delta\varphi_n, \tag{49a}$$
$$\gamma = 2p - 1. \tag{49b}$$

The rotation steps are correlated for $1/2 < p \leq 1$ $(\gamma > 0)$, anticorrelated for $0 \leq p < 1/2$ $(\gamma < 0)$, and uncorrelated for $p = 1/2$ $(\gamma = 0)$. Equation (49a) implies that the correlation function is given by Eq. (28) with

$$B = \Delta\varphi. \tag{50}$$

For small jumps and large n, Eq. (30), the evolution of the probability of horizontal polarization can be approximated by combining Eqs. (21), (31), and (50).

2. Free evolution

In the case of free evolution, Eqs. (C5) and (C6) in Appendix C yield the analytical solution

$$P_h(n) = \frac{1}{2} + \frac{g(r) - g(-r)}{4r}, \tag{51}$$

where $g(r) = (q\cos 2\Delta\varphi + r)(p\cos 2\Delta\varphi + r)^n$ and $r = \sqrt{q^2 - p^2\sin^2 2\Delta\varphi}$.

We now consider special cases of Eq. (51). If $p = q = 1/2$ we get the Markovian exponential decay given by Eq. (47). For $4\Delta\varphi^2 \ll 1, q^2/p^2$ one obtains $r \approx q - 2p^2\Delta\varphi^2/q \approx q$ and Eq. (51) reduces to Eq. (22) with $R_0 = p(\Delta\varphi)^2/q\tau_{\mathrm{r}}$, which, in view of Eqs. (49b) and (50), reduces to the formula for R in (31) with $\theta = 1$. The corresponding behavior can be interpreted as a random sequence of "independent" rotations of the length $\Delta\varphi p/q$, each rotation taking place after p/q steps. Using the same arguments as for the derivation of Eq. (47), we arrive at

$$P_h(n) \approx \frac{1}{2} + \frac{1}{2}\left[\cos\left(2\Delta\varphi\frac{p}{q}\right)\right]^{\frac{qn}{p}}. \tag{52}$$

Finally, for $p = 1$ (i.e., $\gamma = 1$) the rotation steps are infinitely long – the exponential approximation is never true and Eq. (51) yields the "Rabi oscillations" (1).

3. "Impulsive" (projective) measurements.

Let us now assume that during each round-trip the polarization state of the photon is projected on the horizontal polarization ($\theta = 0$ in (5)). The total probability of the horizontal polarization is given by Eq. (2), independently of the memory of the rotator.

Since for $p > q$ this exponential decay is slower than the non-Markovian decay (52) [or, more exactly, (51)], there exists a region of n values for which the probability of the horizontal polarization is larger with the projection than without it. (The region is finite because (52) and (51) decay towards 1/2 whereas (2) decays to zero.)

Figures 4(a)-(c) represent the discussed model. Figure 4(a) exhibits the QZE. In Fig. 4(b) we see the case of exponential Markovian decay ($p = 1/2$) for which the QZE does not take place. It differs from the uninterrupted evolution, which tends to equilibrium of the population. Figure 4(c) exhibits the AZE for $p < 1/2$.

Generally, the smaller $\Delta\varphi$ the slower are thermalization and decay. For $1/2 < p < 1$ (corresponding to $0 < \gamma < 1$ in (49b); cf. Sec. V A) the larger p the larger is decay inhibition by the measurements (QZE), whereas for $0 < p < 1/2$ ($-1 < \gamma < 0$) the opposite trend, i.e., decay acceleration (AZE) occurs. The generic properties of the QZE and the AZE clearly follow from this very simple model.

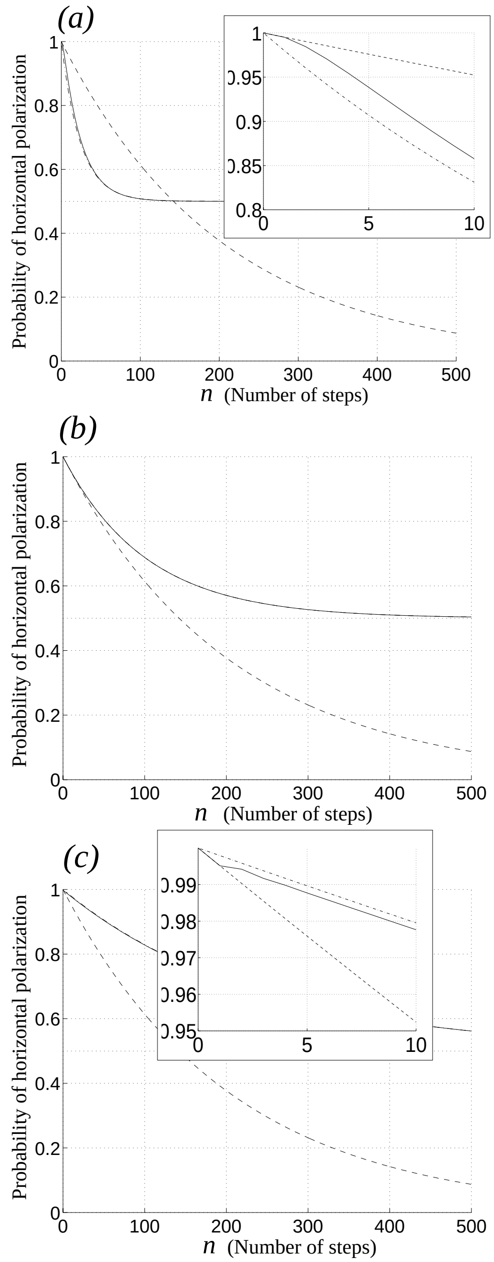

FIG. 4. Evolution of the probability of horizontal polarization for $\Delta\varphi = 4°$: (a) $p = 0.8$, (b) $p = 0.5$, (c) $p = 0.3$, as a function of the number of round trips n. Inset (a): same parameters as (a) but for smaller n. Inset (c): same parameters as (c) but for smaller n. Solid line, the probability of the horizontal polarization without measurements, Eq. (51); dash-dotted line, the exponential approximation (52); dashed line, decay with perfect measurements (complete absorption), as in Eq. (2).

VII. CONCLUSIONS

We have obtained the general conditions of decay inhibition or acceleration via polarization measurements. This has been accomplished by expressing the decay rate as a convolution of two functions: (i) the fluctuation spectrum of the random polarization-rotation rate, which is analogous to the spectrum of the infinite-temperature reservoir; (ii) the Fourier transform of the measurement-induced polarization dephasing, which is determined by the absorber transparency. Inhibition of the decay by frequent measurements (the QZE) has been shown to occur when the reservoir spectrum is peaked around zero frequency, which corresponds to correlated polarization-angle jumps. However, if the jumps are anti-correlated, the reservoir spectrum is split into two peaks, with a dip at zero frequency. Then one should obtain the quantum AZE, i.e., an increase of the decay rate as the rate of effective measurements increases.

In [12] only *direct* measurement schemes were considered (Cook's [6] scheme with pulsed or cw measuring field), whereas here we have considered an *indirect destructive* measurement scheme of the polarization state. Our analysis confirms that the general formula in [12] holds for *direct and indirect* measurement schemes, irrespective of their differences, provided they approximately yield projections on the measured state. The general formula in [12] for the effect of frequent measurements on decay was claimed to hold not just for ideal measurements, i.e., instantaneous projections, but also for continuous measurements. Generally, measurements can be ineffective (i.e., producing projections with a probability less than one) [23]. Here we have considered a comprehensive measurement model, which encompasses the various types of measurements: ideal, ineffective (impulsive), and continuous. The present analysis corroborates the generality of the formula in [12] and provides unifying expressions for the measurement-induced broadening function $F(\omega)$ [Eq. (35)] and the effective measurement rate ν [Eq. (38)].

The significance of the present analysis lies in the physical simplicity of the model and its experimental realizability, as well as in the ability to control the reservoir spectrum (or memory time) and measurement rate by adjustment of the Pockels-cell modulation (on a ns scale) and the PBS transparency, respectively. It allows us to test theoretically (and, hopefully, experimentally in the future) the general conclusions from Ref. [12] for the Zeno and anti-Zeno effects.

ACKNOWLEDGMENTS

This work was supported by ISF (G. K.), the Ministry of Absorption (A. K.) and Minerva (T. O. and G. K.).

APPENDIX A: DERIVATION OF THE MASTER EQUATIONS AND THEIR VALIDITY CONDITIONS

To derive the equation for the average solution of Eq. (8) we use the cumulant expansion technique [25,26]. Assuming, for simplicity, that the odd moments of $\mu(t)$ vanish, we obtain

$$\frac{d\bar{Q}}{dt} = A(t)\bar{Q} + \int_0^t dt' K(t,t')\bar{Q}(t'), \tag{A1}$$

where

$$K(t,t') = \Theta_2(t,t') + \int_{t'}^t dt_1 \int_{t'}^{t_1} dt_2 \Theta_4(t,t_1,t_2,t') + \ldots. \tag{A2}$$

Here the totally ordered cumulants are

$$\Theta_2(t,t') = k(t,t')CU(t,t')C, \tag{A3a}$$

$$\Theta_4(t,t_1,t_2,t') = M_4(t,t_1,t_2,t')CU(t,t_1)CU(t_1,t_2) \times CU(t_2,t')C \tag{A3b}$$

where $U(t,t') = \exp[\int_{t'}^t d\tau A(\tau)]$ and

$$M_4(t,t_1,t_2,t') = \langle\mu(t)\mu(t_1)\mu(t_2)\mu(t')\rangle - k(t,t_1)k(t_2,t') \tag{A4}$$

is the the fourth cumulant of $\mu(t)$, the angular brackets denoting the average.

Truncating the cumulant expansion (A2) at the second order, one obtains the following integro-differential equations for the elements of the polarization tensor,

$$\frac{d\bar{P}_h}{dt} = -2\int_0^t dt' k(t,t')\theta(t,t')[\bar{P}_h(t') - \bar{P}_v(t')], \tag{A5a}$$

$$\frac{d\bar{P}_v}{dt} = 2\int_0^t dt' k(t,t')\theta(t,t')[\bar{P}_h(t') - \bar{P}_v(t')] - \Gamma(t)\bar{P}_v, \tag{A5b}$$

$$\frac{d\bar{u}}{dt} = -\frac{\Gamma(t)}{2}\bar{u} - 2\int_0^t dt' k(t,t')[1+\theta^2(t,t')]\bar{u}(t'). \tag{A5c}$$

Note that Eqs. (A5a) and (A5b) for the polarization probabilities are uncoupled from Eq. (A5c) for the coherence.

We assume that $\bar{P}_h(t')$ and $\bar{P}_v(t')$ are slowly changing with respect to the integral kernel. Then one can set $\bar{P}_{h,v}(t') \approx \bar{P}_{h,v}(t)$ in Eqs. (A5a) and (A5b), yielding Eqs. (10).

Consider now the validity conditions of Eqs. (10) for the case when $\mu(t)$ is a continuous-time stationary random process and $\Gamma(t) = \Gamma_0$ is constant. First, the condition to transform Eqs. (A5a) and (A5b) into Eqs. (10) can be seen to be

$$R \ll \Gamma_{\mathrm{R}} + \Gamma_0. \tag{A6}$$

Second, one can show that taking into account the fourth cumulant (A3b) in Eqs. (A1) and (A2) amounts to the addition of the term

$$-4\int_{t'}^t dt_1 \int_{t'}^{t_1} dt_2 M_4(t,t_1,t_2,t')\theta(t,t_1)\theta(t_2,t') \times[1+\theta^2(t_1,t_2)] \tag{A7}$$

to the kernel $2k(t,t')\theta(t,t')$ of Eqs. (A5a) and (A5b). The cumulant $M_4(t,t_1,t_2,t')$ tends to zero with the increase of the difference between any two of its arguments. Assuming that all the characteristic decay rates are of the order of $\Gamma_{\rm R}$, one can estimate that the fourth cumulant contributes to the rate R in Eqs. (18) a quantity of the order of $R^2/\Gamma_{\rm R}$. Hence, the fourth cumulant can be neglected under condition (14). Note that condition (14) is stricter than (A6).

APPENDIX B: GENERAL ANALYSIS OF SMALL DISCRETE PHASE JUMPS

The correlation time $\Gamma_{\rm R}^{-1}$ of a random chain $\Delta\varphi_n$ can be defined generally as

$$\Gamma_{\rm R}^{-1} = \frac{\tau_{\rm r}}{B^2}\sum_{n=0}^{\infty} K_n. \tag{B1}$$

Assuming the validity of condition (14), the solution of Eqs. (10) with the initial conditions (20) can be obtained at $t = n\tau_{\rm r}$ in the form

$$\begin{pmatrix} \bar{P}_h(t) \\ \bar{P}_v(t) \end{pmatrix} = \exp\left[\begin{pmatrix} -W_n & W_n \\ W_n & -W_n - \Gamma_0 t \end{pmatrix}\right]\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \tag{B2}$$

where $W_n = \int_0^t dt' R(t')$.

To prove Eq. (B2) we cast Eqs. (10) in the form $\dot{q} = L(t)q$, where

$$q(t) = \begin{pmatrix} \bar{P}_h(t) \\ \bar{P}_v(t) \end{pmatrix}, \qquad L(t) = \begin{pmatrix} -R(t) & R(t) \\ R(t) & -R(t) - \Gamma(t) \end{pmatrix}. \tag{B3}$$

Hence $q(t) = U(t)q(0)$, where $U(t) = T\exp[\int_0^t dt' L(t')]$, T being the time-ordering operator. Note that $L(t) \approx L(\infty) = L$ for $t \gg \Gamma_{\rm R}^{-1}$. Consider two cases. Let, first, $\Gamma_0 \ll \Gamma_{\rm R}$. We want to show that T can be omitted in the expression for $U(t)$, resulting in the coincidence of $U(t)$ with the matrix in Eq. (B2). For $t \ll (R+\Gamma_0)^{-1}$ one gets $U(t) \approx [1 + \int_0^\tau dt' L(t')] \approx \exp[\int_0^t dt' L(t')]$, whereas for $t \gtrsim (R+\Gamma_0)^{-1}$, choosing time τ such that $\Gamma_{\rm R}^{-1} \ll \tau \ll (R+\Gamma_0)^{-1}$, one obtains $U(t) \approx [1+\int_0^\tau dt' L(t')]e^{L(t-\tau)} \approx \exp[\int_0^t dt' L(t')]$. The second case $\Gamma_0 \gtrsim \Gamma_{\rm R}$ implies, in view of (14), that $\Gamma_0 \gg R$. Then one can set $\bar{P}_v(t) \approx 0$ in Eqs. (10), yielding $\bar{P}_h(t) \approx e^{-W_n}$, which can be shown to follow also from (B2). This finishes the proof of (B2).

The above expression for W_n, with the account of Eqs. (11)-(13) and (5), can be cast as

$$W_n = \sum_{m,m'=1}^{n} K_{mm'}\theta^{|m-m'|} \tag{B4}$$

or, for a stationary process,

$$W_n = nB^2 + 2\sum_{m=1}^{n-1}\sum_{m'=1}^{m} K_{m'}\theta^{m'}. \tag{B5}$$

In the case (28), the calculation of (B5) shows that, under condition (30a),

$$W_n \approx nB^2\frac{1+\gamma\theta}{1-\gamma\theta} - \frac{2B^2\gamma\theta}{(1-\gamma\theta)^2}. \tag{B6}$$

For

$$B^2 \ll (1-\gamma\theta)^2 \tag{B7}$$

the second term in the right-hand side of Eq. (B6) can be neglected and Eq. (B2) yields (21) with the definitions (31).

For an arbitrary correlation function K_n one can show with the help of Eq. (B5) that Eq. (31) for R should be substituted by

$$R = \frac{1}{\tau_{\rm r}}\left(B^2 + 2\sum_{n=1}^{\infty} K_n\theta^n\right) \tag{B8}$$

or, equivalently, by Eq. (32).

The validity conditions of the above results are: $t \gg \Gamma_{\rm R}^{-1}$ [roughly corresponding to inequality (30a)], the inequalities (14) and $B^2 \ll 1$. In the case (28) the latter two inequalities can be combined to obtain Eq. (30b), which is generally stricter than (B7).

Equation (32) can be rewritten in the form (33) with

$$F(\omega) = \frac{\tau_{\rm r}}{2\pi}\sum_{n=-\infty}^{\infty}\theta^{|n|}e^{-in\omega\tau_{\rm r}} \tag{B9}$$

and

$$G(\omega) = \frac{1}{2\pi\tau_{\rm r}}\sum_{n=-\infty}^{\infty} K_n e^{in\omega\tau_{\rm r}} = \frac{1}{2\pi\tau_{\rm r}}\left(B^2 + 4\sum_{n=1}^{\infty} K_n\cos n\omega\tau_{\rm r}\right). \tag{B10}$$

The function $F(\omega)$ (B9) is normalized to one in the interval $-\pi/\tau_{\rm r} < \omega < \pi/\tau_{\rm r}$. Performing the summation in Eq. (B9) yields Eq. (35). In the special case (28) Eq. (B10) reduces to the closed formula (34).

APPENDIX C: EXACT SOLUTION FOR DISCRETE NON-MARKOVIAN PHASE JUMPS

Here we calculate polarization dephasing for a rather general model of random rotation angles. The average phase factor in Eq. (43) is given generally by

$$\langle e^{2i\varphi_n}\rangle = \sum_{\Delta\varphi_1,\ldots,\Delta\varphi_n} e^{2i\varphi_n}p(\Delta\varphi_1,\ldots,\Delta\varphi_n), \tag{C1}$$

where $p(\Delta\varphi_1,\ldots,\Delta\varphi_n)$ is the joint probability of the random chain $\Delta\varphi_k$.

According to the model of Sec.VI B 1, $\varphi_n = \sum_{k=1}^{n}\Delta\varphi_k$ is a (non-Markovian) sum over a Markovian chain $\Delta\varphi_k$. The average (C1) can be calculated for arbitrary Markovian chains, i.e., chains in which $\Delta\varphi_{k+1}$ depends only on $\Delta\varphi_k$, as follows. In this case

$$p(\Delta\varphi_1,\ldots,\Delta\varphi_n) = P(\Delta\varphi_n,\Delta\varphi_{n-1})\ldots \times P(\Delta\varphi_2,\Delta\varphi_1)p_0(\Delta\varphi_1). \qquad \text{(C2)}$$

Here $p_0(\Delta\varphi_1)$ is the unconditional probability and $P(\Delta\varphi_k,\Delta\varphi_{k-1})$ is a conditional probability. Combining Eqs. (C2), (C1) and (43), one can obtain that

$$P_h(n) = \frac{1}{2} + \frac{1}{2}\mathrm{Re}\sum_{\Delta\varphi_{n+1}} f_{n+1}(\Delta\varphi_{n+1}). \qquad \text{(C3)}$$

The quantity $f_{n+1}(\Delta\varphi_{n+1})$ is defined by the following iterative relation, which is convenient for numerical calculations,

$$f_1(\Delta\varphi_1) = p_0(\Delta\varphi_1),$$
$$f_{n+1}(\Delta\varphi_{n+1}) = \sum_{\Delta\varphi_n} P(\Delta\varphi_{n+1},\Delta\varphi_n)e^{2i\Delta\varphi_n} f_n(\Delta\varphi_n). \quad \text{(C4)}$$

The above solution can be conveniently written in a matrix form. Assuming that the random variable $\Delta\varphi_k$ can assume the values $\delta\varphi_i$ $(i = 1,2,\ldots)$, we represent $p_0(\Delta\varphi_k)$ by a column vector p_0 with the components $p_0(\delta\varphi_i)$. The conditional probability is represented by the matrix P with $P_{ij} = P(\delta\varphi_i,\delta\varphi_j)$. Then Eq. (C3) and (C4) yield

$$P_h(n) = \frac{1}{2} + \frac{1}{2}\mathrm{Re}[u(P\Phi)^n p_0], \qquad \text{(C5)}$$

where u is the row vector with the components equal to 1 and Φ is a diagonal matrix with $\Phi_{jj} = e^{2i\delta\varphi_j}$.

For the non-Markovian model of Sec.VI B 1

$$p_0 = \frac{1}{2}\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad P = \begin{pmatrix} p & q \\ q & p \end{pmatrix}, \quad \Phi = \begin{pmatrix} e^{2i\Delta\varphi} & 0 \\ 0 & e^{-2i\Delta\varphi} \end{pmatrix}. \qquad \text{(C6)}$$

[1] L. A. Khalfin, ZhETF Pis. Red. **8**, 106 (1968) [JETP Lett. **8**, 65 (1968)].
[2] L. Fonda, G. C. Ghirardi, A. Rimini, and T. Weber, Nuov. Cim. **15A**, 689 (1973).
[3] B. Misra and E. C. G. Sudarshan, J. Math. Phys. **18**, 756-763 (1977).
[4] W. M. Itano, D. J. Heinzen, J. J. Bollinger, and D. J. Wineland, Phys. Rev. A **41**, 2295 (1990).
[5] E. Joos, Phys. Rev. A **29**, 1626 (1984).
[6] R. J. Cook, Phys. Scr. **T21**, 49 (1988).
[7] V. Frerichs and A. Schenzle, Phys. Rev. A **44**, 1962-1968 (1991).
[8] S. R. Wilkinson, C. S. Bharucha, M. S. Fischer, K. W. Madison, P. R. Morrow, Q. Niu, B. Sundaram and M. G. Raizen, Nature (London) **387**, 575 (1997); Q. Niu and M. G. Raizen, Phys. Rev. Lett. **80**, 3491 (1998).
[9] B. Elattari and S. A. Gurvitz, Phys. Rev. Lett. **84**, 2047 (2000).
[10] J. J. Sakurai, *Modern Quantum Mechanics* (Addison-Wesley, Reading, 1994), pp. 484-486.
[11] L. S. Schulman, Phys. Rev. A **57**, 1509 (1998).
[12] A.G. Kofman and G. Kurizki, Nature (London) **405**, 546 (2000).
[13] The AZE was discovered by A.G. Kofman and G. Kurizki, Phys. Rev. A **54**, R3750 (1996), for the case of excited-atom decay in an off-resonant cavity. Related effects have been noted in proton decay by A. M. Lane, Phys. Lett. **99A**, 359 (1983) and more recently in photodetachment by M. Lewenstein and K. Rzążevski, Phys. Rev. A **61**, 022105 (2000).
[14] G. Harel, A.G. Kofman, A. Kozhekin and G. Kurizki, Opt. Exp. **2**, 355 (1998).
[15] M. Lewenstein and K. Rzążewski, Phys. Rev. A **61**, 022105 (2000).
[16] A. Peres, Am. J. Phys. **48**, 931 (1980).
[17] A. D. Panov, Ann. Phys. **249**, 1 (1996); Phys. Lett **260**, 441 (1999); J. Audretsch, M. B. Mensky, and A. D. Panov, Phys. Lett **261**, 44 (1999).
[18] This is a modification of the setup used by P.G. Kwiat, Physica Scripta **T76**, 115 (1998).
[19] A. Elitzur and L. Vaidman, Found. Phys. **23**, 987 (1993).
[20] P.G. Kwiat, H. Weinfurter, T. Herzog, A. Zeilinger, and M. A. Kasevich, Phys. Rev. Lett. **74**, 4763 (1995).
[21] P.G. Kwiat, A. G. White, J. R. Mitchell, O. Nairz, G. Weihs, H. Weinfurter, and A. Zeilinger, Phys. Rev. Lett. **83**, 4725 (1999).
[22] G. J. Milburn, J. Opt. Soc. Am. B **5**, 1317 (1988).
[23] A. Peres and A. Ron, Phys. Rev. A **42**, 5720 (1990).
[24] L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields* (Pergamon Press, Oxford, 1975), Sec. 50.
[25] R. H. Terwiel, Physica **74**, 248 (1974).
[26] A. G. Kofman, R. Zaibel, A. M. Levine, and Y. Prior, Phys. Rev. A **41**, 6434 (1990).
[27] A. G. Kofman and G. Kurizki, acta phys. slovaca **49**, 541 (1999).
[28] N. G. van Kampen, *Stochastic Processes in Physics and Chemistry* (North-Holland, Amsterdam, 1981), sec. IV.5.